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ANALYTICAL TEST CASE'S FOR NEUTRON AND RADIATION TRANSPORT CODES

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ABSTRACT

This paper collects together all the methods I know of by which analytical solutions to the transport equation may be obtained.

A continuing need of the numerical analyst is a supply of analytical solutions that may be used as benchmarks against which computed results may be checked. Listed below are those methods that seem useful for checking neutron and radiation transport codes.

1. Approximate solutions. Of course this is a vast subject, and not really what I want to discuss here, but they should be kept in mind. Diffusion theory, collision probabilities, variational methods, etc., can give very accurate results when applied to the right problems.

2. Streaming in a Vacuum. See Case, deHoffmann and Placzek.

3. No Scattering or Fission. In this case the transport equation is just a first order partial differential equation with a known source that is easily solved by characteristics. Energy enters only as a parameter. The straight-forward approach is to use a Green's function for an anisotropic point source in an infinite medium (for the time independent case, see p. 38 of the book by Case and Zweifel). The only restriction on sources, geometry and cross sections is that you must be able to do the integrations.

4. Infinite Space and Half-Space Problems. Solutions to these problems are expressed in terms of integrals of known functions. They may be found in both the references mentioned. They aren't very useful as test cases because the codes can't handle infinite space. Energy-dependent problems have been solved by making various assumptions about the kernel (see the book by Williams), but the "solutions" are a long way from numbers. This is even more true for time-dependent solutions to infinite space and half-space problems.

5. Finite Slab and Sphere, Case's Method. These "solutions" are not solutions at all, but are merely a reformulation of the problem. The only practical use to which Case's method has been put has been to generate asymptotic (away from boundaries) solutions.

6. Particular Solutions. Ozisik and Siewert (Nucl. Sci. Eng., 40, 491 (1970)) have obtained particular solutions to the equation

$$\mu \frac{\partial}{\partial x} \psi + \psi = \frac{c}{2} \int_{-1}^{+1} (1 + b\mu\mu')\psi \, d\mu' + S(x,\mu)$$

for a number of source terms S. The limitation to plane infinite systems coupled with the fact that they are often negative or singular limits their usefulness.

7. Some Miscellaneous Methods. In Appendix I of the book by Case and Zweifel, and in a very nice paper by Gibbs (J. Math. Phys., 10, 875 (1969)) a formal extension of Case's method has been made to regions bounded by the level surfaces of coordinate systems for which the Helmholtz equation is separable. I doubt that this is of any computational value. The same is probably true of the work using generalized analytic functions (Ferziger's paper at the VPI conference) and three dimensional normal modes (Bareiss, ANL-6941, ANL-7328, SIAM Conf. on Neutron Transport Theory, p. 37). Bareiss has expressed the hope that his three dimensional normal modes could be used for testing codes. He is unable to solve a given boundary value problem, but his modes are still solutions to some problem. However, I see no advantage in using Bareiss's solutions over the method proposed below.

8. Finding a Problem for a Given Solution. The above discussion would lead one to believe that there is not much in the way of analytical solutions to test codes. This is true only if one insists on specifying the problem to be solved beforehand. An alternative is to specify the solution beforehand. The first step is to write down the transport equation in some coordinate system for infinite space.

$$\frac{1}{v} \frac{\partial \psi}{\partial t} + \Omega \cdot \nabla \psi + \sigma \psi = \frac{c\sigma}{4\pi} \int_{\Omega} K(\Omega' \rightarrow \Omega) \psi \, d\Omega' + S$$

Now substitute a solution for ψ into the equation; for the energy-dependent case and reasonable solutions, the gradiant and integration over Ω operations are easy. Even the energy-dependent case can be done if the kernel is simple enough. The result is an analytic expression for S. It is this S which would produce the solution for ψ that you started with for the infinite space problem. The transition to solutions in bounded geometries is easy: the same S results in the same ψ in a finite problem if we simply impose the (known) ψ as a boundary condition on the surface for incoming directions (surface source). A little care is necessary if the code has a negative flux fix-up: one must choose a region in which both S and ψ are positive, but this is usually easy to do. Also it is obvious that the code must be general enough to handle an anisotropic source and surface source (time-dependent too, for time-dependent problems). This procedure has the advantage that one can directly test the ability of a code to calculate solutions that have a specified behavior in a specified region (say rapidly varying near a boundary). For purpose of illustration, suppose one has a slab extending from 0 to a. The internal source is

$$S(x,\mu) = (2 - \frac{5c}{3})\cos x - 2(\sin x)\mu - (\cos x)\mu^2 + (\sin x)\mu^3;$$

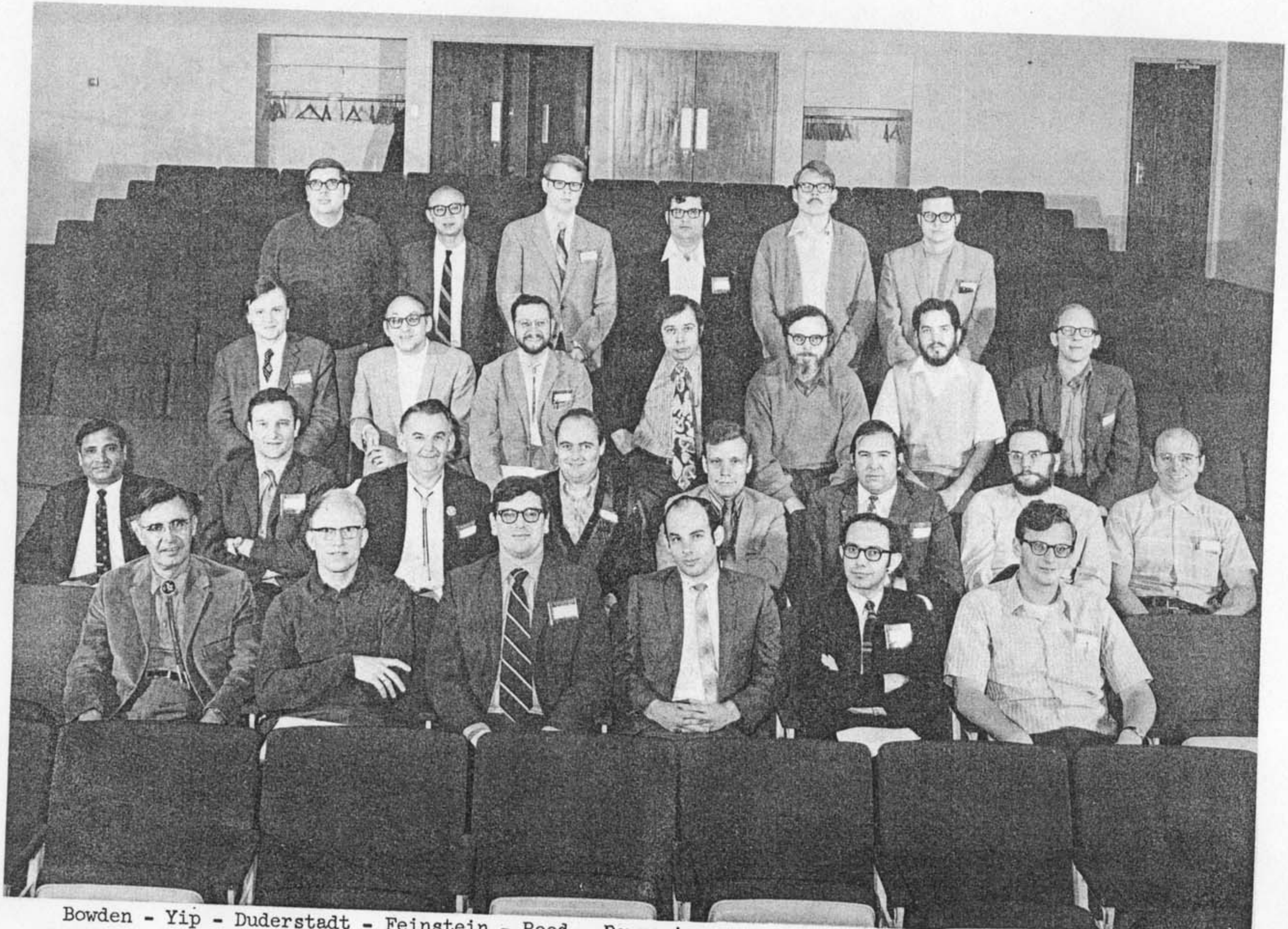
the left boundary condition is reflecting and the right boundary condition is vacuum plus the surface source

$$\psi = (2 - \mu^2)\cos a.$$

The problem is time-independent and $\sigma = 1$. Then the solution for ψ is

$$\psi = (2 - \mu^2)\cos x.$$

One can make the internal source go away by using asymptotic solutions, as in item (7) above, but you're still stuck with the surface source. Even the surface source could be made small (in its effects on the scalar flux) for many problems by choosing for ψ the solution to a problem with no surface source in asymptotic diffusion theory (i.e., using Davison's boundary conditions, see Chapter 8 of his book).



Bowden - Yip - Duderstadt - Feinstein - Reed - Pomraning
Nicolaenko - Aronson - Skumanich - Siewert - Zweifel - Gibbs - Lathrop
Samaddar - Atkinson - Carlson - Dorning - Burniston - Hunt - Ferziger - Hendry
Wing - Kaper - Shultis - Nestell - Abu-Shumays - Leonard
ıvited Speakers not in picture: Allen, Bareiss, Case, Corngold, Erdmann, Gelbard, Thurber, Yan, and Lingus)